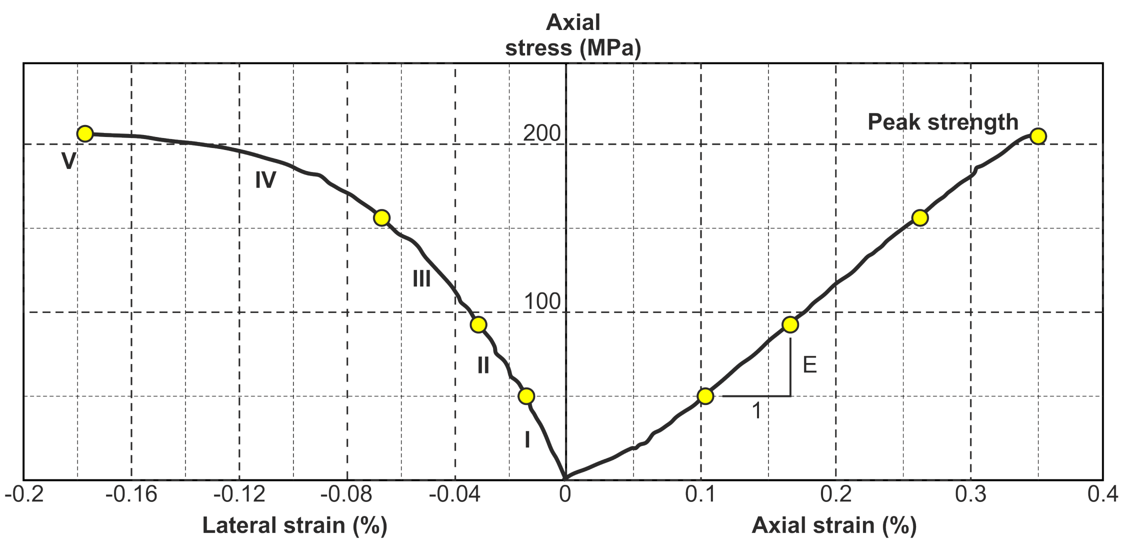
Practice exam : answer model

Question 1 (25 points)

a Describe the stress - strain curves of Lac du Bonnet granite in terms physical processes occurring in the sample. Be specific and refer to the following diagram, which you can annotate. [**7.5 points**]



As the rock sample is progressively loaded in the axial direction, it deforms axially and radially. The stress-strain curve for the Lac du Bonnet granite can be divided into four regions (marked in the above graph) :

- Region I ($\sigma_a = 0 - 50$ MPa) : the axial strain-axial stress curve is concave upwards. This region corresponds to the closure of existing microcracks in the sample.
- Region II ($\sigma_a = 50 - 95$ MPa) : the behaviour of the rock sample is linear elastic (both axially and radially). The elastic properties of a rock sample can be determined from this portion of the stress-strain curves.
- Region III ($\sigma_a = 95 - 160$ MPa) : the lateral strain deviates from linearity (dilation). The axial stress-strain curve is nearly elastic. Significant acoustic emissions can be recorded, corresponding to an increase in micro-cracking.
- Region IV ($\sigma_a = 160 - 205$ MPa) : the stress - strain behaviour deviates from linearity. The rock undergoes a rapid acceleration of micro-cracking events and volume increase.
- V ($\sigma_a = 205$ MPa) : the peak strength σ_c is the maximum stress that a rock specimen can sustain. It marks the beginning of the post-peak behaviour.

b Determine Young's modulus and Poisson's ratio of Lac du Bonnet granite. [**3 points**]

$$E = \frac{\Delta\sigma}{\Delta\varepsilon_{axial}} = \frac{95 - 50}{0.17 \cdot 10^{-2} - 0.10 \cdot 10^{-2}} \text{ MPa} = 64286 \text{ MPa} = 64.29 \text{ GPa}$$

$$\nu = \frac{-\Delta\varepsilon_{radial}}{\Delta\varepsilon_{axial}} = \frac{-(0.032 \cdot 10^{-2} - 0.014 \cdot 10^{-2})}{0.17 \cdot 10^{-2} - 0.10 \cdot 10^{-2}} = 0.26$$

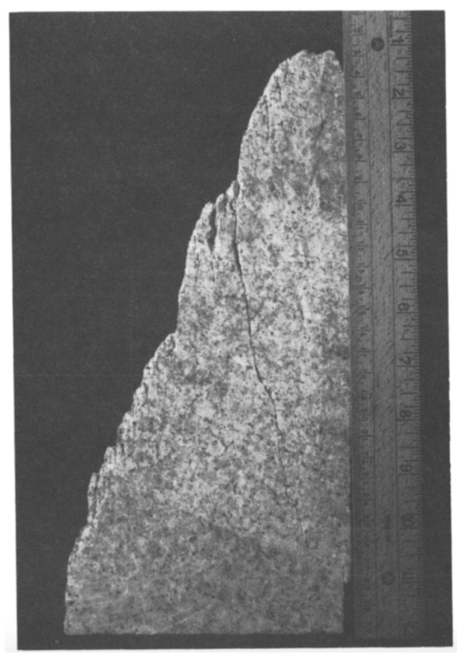
c Determine the uniaxial compressive strength of Lac du Bonnet granite. [**1.5 point**]

$$\sigma_c = 205 \text{ MPa}$$

d Which class of material does the of Lac du Bonnet granite belong to ? [**1 point**]

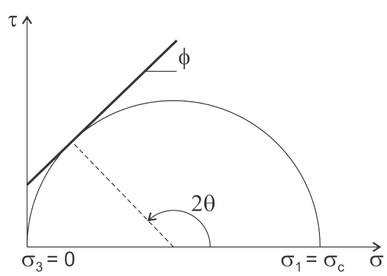
Class II, brittle behaviour

e After failure, Martin and Chandler took a picture of the sample. They recorded that the specimen failed along a single fracture. Which information does the following picture give about the strength of Lac du Bonnet granite ? Give a qualitative and quantitative answer. [**3 points**]



Units on the right of the ruler are cm.

The orientation of the failure plane gives an indication on where the Mohr circle is tangent to the failure line (i.e. where it "hits" the failure criterion), so that the friction angle of the rock can be determined.



The angle θ between the horizontal and the fracture plane is equal to :

$$\theta = 67°$$

so that the friction angle is given by :

$$\phi = 2 \cdot 67° - 90° = 44°$$

(assuming Mohr-Coulomb failure criterion).

f Propose a failure criterion of the form $\sigma_1 = a\sigma_3 + b$ for Lac du Bonnet granite. [**4 points**]

The Mohr-Coulomb failure criterion written in its principal stress form takes the form :

$$\sigma_1 = \frac{1+\sin\phi}{1-\sin\phi}\sigma_3 + \sigma_c$$

Knowing the friction angle $\phi = 44°$ and the uniaxial compressive strength $\sigma_c = 205$ MPa, it comes $a = 5.55$ and $b = 205$ MPa.

<u>Remark</u> : under uniaxial compression, $\sigma_3 = 0$ and the axial stress at failure is $\sigma_1 = \sigma_c$. It comes $b = \sigma_c$.

g What is the state of stress on the failure plane at failure ? [**4 points**]

The state of stress on the failure plane is obtained using the transformation equations (see formulae sheet) :

$$\sigma_n = \sigma_3 \sin^2\theta + \sigma_1 \cos^2\theta - 2\tau_{13}\sin\theta\cos\theta = \sigma_c \cos^2\theta$$

$$\tau = \tau_{13}\left(\cos^2\theta - \sin^2\theta\right) - (\sigma_x - \sigma_y)\sin\theta\cos\theta = \sigma_c \sin\theta\cos\theta$$

with $\sigma_c = 205$ MPa and $\theta = 67°$.

Accordingly, the normal and shear stresses on the failure plane at failure are equal to :

$$\sigma_n = 31.3 \text{ MPa}$$

$$\tau = 73.7 \text{ MPa}$$

h As part of its research program on the geological disposal of high-level radioactive waste, the Canadian Nuclear Fuel Waste Management Program has constructed an Underground Research Laboratory in the Lac du Bonnet batholith. High-level radioactive waste contains radionuclides which are strongly heat emitting. Heating arising from the canisters will induce significant temperature changes in the host formation. What effects is temperature likely to have on the mechanical behaviour of Lac du Bonnet granite ? [**1 point**]

— An increase in temperature is likely to reduce the elastic modulus and compressive strength.

— An increase in temperature is likely to increase the ductility in the post-peak region.

Question 2 (25 points)

a Calculate the vertical and horizontal *in situ* stresses before excavation of the shaft. Consider the theory of elasticity and assume conditions of uniaxial strain in the vertical direction with zero lateral strain. [**4 points**]

The vertical stress is given by the weight of the overburden :

$$\sigma_v = \gamma_r \cdot z \Rightarrow \sigma_v(500 \text{ m}) = 26 \text{ kN/m}^3 \cdot 500 \text{ m} = 13000 \text{ kPa} = 13 \text{ MPa}$$

The theory of elasticity gives us :

$$\sigma_h = \frac{\nu}{1-\nu} \cdot \sigma_v \Rightarrow \sigma_h(500 \text{ m}) = 0.54 \cdot 13 \text{ MPa} = 7 \text{ MPa}$$

b Develop a mathematical expression of the shear stress acting on the fault plane as a function of the angle θ between the normal stress acting on the fault plane σ_n and the radial stress σ_r, and of the distance between the shaft axis O and the fault $(L+R)$. [**10 points**]

(a) Using the transformation equations (see formulae sheet), the shear stress on the fault plane can be expressed as :

$$\tau = -(\sigma_r - \sigma_\theta) \sin\theta \cos\theta$$

where σ_r is the radial stress, σ_θ is the tangential stress and θ is the angle between σ_n and σ_r.

(b) Kirsch equations (see formulae sheet) give the distribution of stress around a circular opening in an elastic medium :

$$\sigma_r = \sigma_h \left(1 - \frac{1}{\rho^2}\right)$$

$$\sigma_\theta = \sigma_h \left(1 + \frac{1}{\rho^2}\right)$$

where $\rho = r/R$, r being the radial distance and R the shaft radius.

(c) Combining (a) and (b) gives :

$$\tau = -(\sigma_r - \sigma_\theta) \sin\theta \cos\theta = \sigma_h \frac{2}{\rho^2} \sin\theta \cos\theta = \sigma_h \frac{R^2}{r^2} \sin 2\theta$$

(d) The distance r from the shaft centre to any point of the fault plane can be expressed as a function of θ :

$$r = \frac{r_\perp}{\cos\theta} = \frac{L+R}{\cos\theta}$$

(e) It comes :

$$\tau = \sigma_h \frac{R^2}{(L+R)^2} \cos^2\theta \sin 2\theta$$

c Calculate the value of the angle θ at which the shear stress on the fault plane τ is maximum. [**5 points**]

The derivative of the shear stress with respect to the angle θ is given by

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = \sigma_h \frac{R^2}{(L+R)^2} \left(-2 \cdot \cos\theta \cdot \sin\theta \cdot \sin 2\theta + \cos^2\theta \cdot 2 \cdot \cos 2\theta\right)$$

It is equal to 0 if :

$$-2 \cdot \cos\theta \cdot \sin\theta \cdot \sin 2\theta + \cos^2\theta \cdot 2 \cdot \cos 2\theta = 0$$

$$\iff -\sin\theta \cdot \sin 2\theta + \cos\theta \cdot \cos 2\theta = 0$$

$$\iff \sin\theta \cdot \sin 2\theta = \cos\theta \cdot \cos 2\theta$$

$$\iff \tan\theta = \frac{1}{\tan 2\theta}$$

$$\iff \theta = 30^\circ$$

d Calculate the minimum distance $L + R$ between the shaft axis O and the fault which is required to maintain the elastic stress distribution in the rock mass and avoid slippage along the fault. [**6 points**]

The shear stress on the fault plane is given as :

$$\tau = \sigma_h \frac{R^2}{(L+R)^2} \cos^2\theta \sin 2\theta$$

which should be less than the shear strength of the fault plane, which is equal to 1 MPa.

$$(L_{min} + R)^2 = \sigma_h \frac{R^2}{\tau_{max}} \cos^2\theta \sin 2\theta$$

$$L_{min} = \sqrt{\sigma_h \frac{R^2}{\tau_{max}} \cos^2\theta \sin 2\theta} - R = 3.40 \text{ m}$$

Accordingly, the the minimum distance $L + R$ between the shaft axis O and the fault is equal to :

$$R + L_{min} = 6.40 \text{ m}$$

Question 3 (25 points)

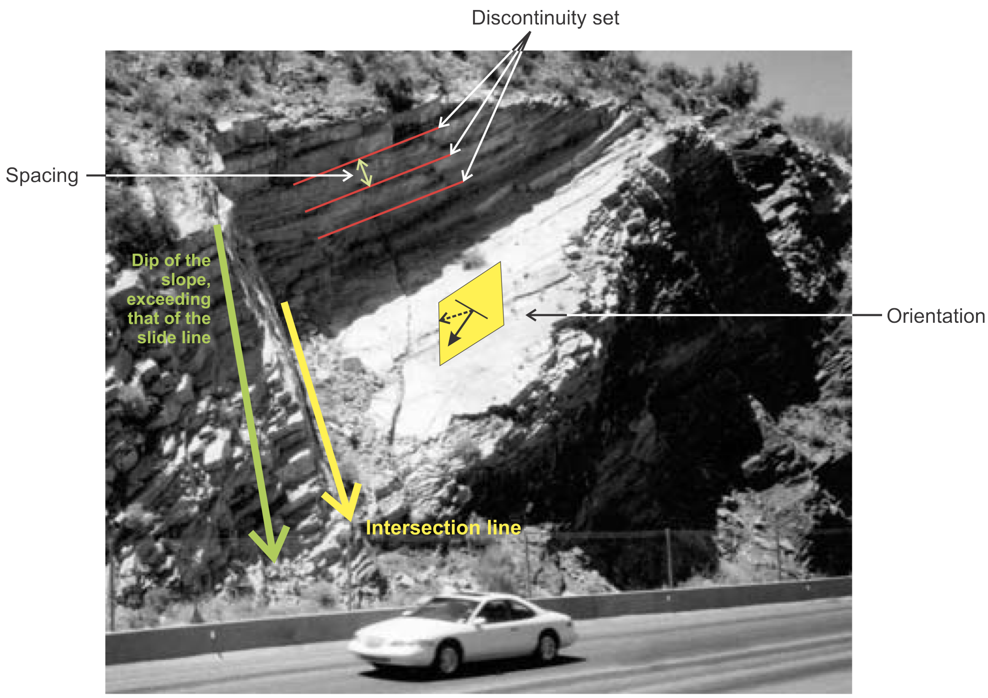
a List 5 geometrical properties of discontinuities. [**5 points**]

- Discontinuity sets
- Persistence
- Orientation
- Spacing and block size
- Aperture

b How does each of these geometrical properties of discontinuities affect the stability of surface excavations (slopes) ? Explain your answers. [**7.5 points**]

- Discontinuity sets : more discontinuity sets provide more possibilities of potential slide planes.
- Persistence : discontinuity sets with a high persistence are generally more critical with respect to stability of surface excavations due to their large areal extent or length.
- Orientation : orientation of joint sets controls the possibility of unstable conditions or excessive deformations.
- Spacing and block size : more joints mean that less average spacing between joints. Joint spacing controls the size of individual rock blocks. It controls the mode of failure and flow.
- Aperture : joint opening is either filled with air and water (open joint) or with infill materials (filled joint). Open or filled joints with large apertures have generally low shear strength. Properties of the filling material affect shear strength and deformability.

c The following picture shows the outcrop of a sedimentary formation near Phoenix, Arizona. Point at 3 geometrical properties of discontinuities in the figure using arrows and label each of them. [**3 points**]



d Which type of slope failure occurred in the outcrop in the above picture ? [**1 point**]

Wedge slope failure

e List the necessary kinematic conditions for this slope failure mechanism to be activated. Highlight two of these conditions on the picture or on a sketch. [**5 points**]

— The potential intersection line must dip out of the slope.

— The dip of the slope must exceed that of the potential slide line.

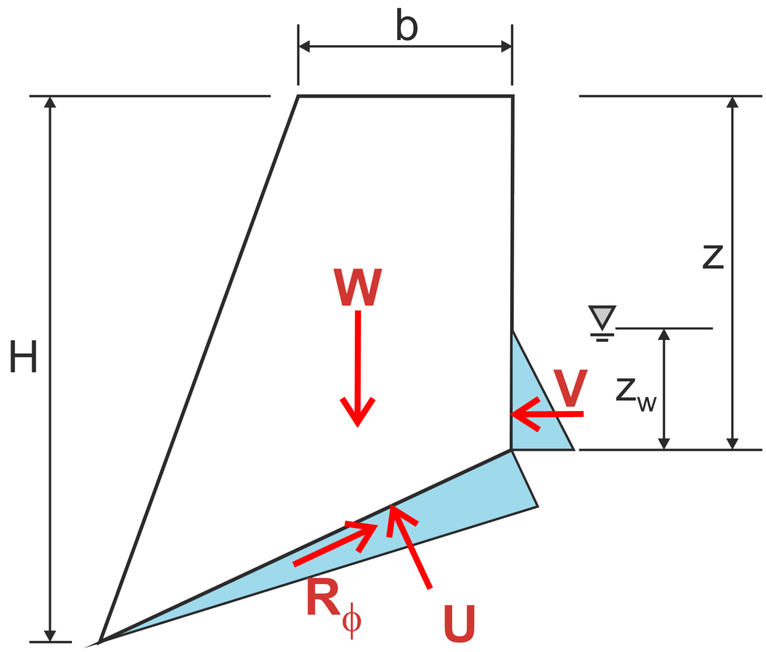
— The dip of the potential slide line is such that the strength of the line is reached.

f What kind of support technique(s) could be implemented to avoid further failure of the slope ? Justify your answer. [**3.5 points**]

Rock bolts (anchors) could be installed across discontinuities (i.e. potential slide surfaces) up to the sound rock. Pre-tensioning would increase the normal stress acting on the discontinuity planes and modify the shear stress, thereby increasing the factor of safety.

Question 4 (25 points)

a List, label, represent and give the mathematical expression the forces acting on the block in the following figure. [**8 points**]



Note : a 1-m thick slice of the slope is considered.

— W : weight of the rock block

$$W = \gamma_r \left[\left(b + \frac{H-z}{\tan \psi_p} \right) \frac{H}{2} - \frac{1}{2} \frac{(H-z)^2}{\tan \psi_p} \right]$$

— U : resultant force of water pressure on the sliding surface

$$U = \frac{1}{2} \gamma_w z_w \frac{H-z}{\sin \psi_p}$$

— V : resultant force of water pressure in the tension crack

$$V = \frac{1}{2} \gamma_w z_w^2$$

— R_ϕ : resisting force due to friction along the slide plane

$$R_\phi = (W \cos \psi_p - U - V \sin \psi_p) \tan \phi$$

b Develop a mathematical expression of forces resisting failure along the slide plane. [**2 points**]

$$F_{resisting} = R_\phi$$

c Develop a mathematical expression of forces driving failure along the slide plane. [**2 points**]

$$F_{driving} = W \sin \psi_p + V \cos \psi_p$$

d Give the mathematical expression and calculate the factor of safety against planar failure along the slide plane if the rock mass is dry. [**4 points**]

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{(W \cos \psi_p - U - V \sin \psi_p) \tan \phi}{W \sin \psi_p + V \cos \psi_p}$$

In case of dry rock mass ($U = 0$ and $V = 0$), the factor of safety is :

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{W \cos \psi_p \tan \phi}{W \sin \psi_p} = \frac{\cos \psi_p \tan \phi}{\sin \psi_p} = \frac{\tan \phi}{\tan \psi_p} = 1.56$$

e Is the dry rock mass stable or not ? [**1 point**]
Yes.

f Would removing the unstable rock by blasting improve the stability of the dry slope ? Argue. [**2 points**]

— The factor of safety does not depend on the weight of the block. However, removing the unstable rock would actually cancel the risk and *a priori* improve the stability of the dry slope.

— However, the risk might be cancelled temporally as new tension cracks are likely to form.

g Calculate the factor of safety against planar failure along the slide plane if $z_w = 11$ m. Assume $\gamma_w = 10$ kN/m^3. [**5 points**]

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{(W\cos\psi_p - U - V\sin\psi_p)\tan\phi}{W\sin\psi_p + V\cos\psi_p}$$

with :

$$W = \gamma_r\left[\left(b + \frac{H-z}{\tan\psi_p}\right)\frac{H}{2} - \frac{1}{2}\frac{(H-z)^2}{\tan\psi_p}\right] = 4552 \text{ kN}$$

$$U = \frac{1}{2}\gamma_w z_w \frac{H-z}{\sin\psi_p} = 1004 \text{ kN}$$

$$V = \frac{1}{2}\gamma_w z_w^2 = 605 \text{ kN}$$

It comes :

$$FoS = 0.84$$

h Is the wet rock mass with $z_w = 11$ m stable or not ? [**1 point**]
No.